GOLDEN CODE AND GOLDEN Coded BEAMFORMING
IN MIMO

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Abstract: In MIMO networks, the spatial degrees of freedom offered by multiple antennas can be advantageously exploited to enhance the system capacity. For this scheduling of multiple users is done to simultaneously share the spatial channel. The Golden code (GC) was proposed when no binary outer code is applied at the Transmitter in 2×2 MIMO systems. This code achieves full rate, full diversity and the Diversity Multiplexing Tradeoff hence use of this code is optimal. We propose in this paper, implementation of Golden code in Multiple-Input-Multiple-Output (MIMO) system.

Keywords: Multiple-input multiple-output (MIMO), Golden code (GC), Spatial Division Multiplexing (SDM), Space Time Block Coding (STBC), Golden Coded Multiple Beamforming (GCMB).

1 INTRODUCTION

Wireless communication using multiple-input multiple-output (MIMO) systems enables increased spectral efficiency for a given total transmit power. Increased capacity is achieved by introducing additional spatial channels that are exploited by using space-time coding. These factors include channel complexity, external interference, and channel estimation error. The ‘multichannel’ term indicates that the receiver incorporates multiple antennas by using space-time-frequency adaptive processing. In radio, multiple-input and multiple-output, or MIMO (commonly pronounced my-moh or me-moh), is the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. The terms input and output refer to the radio channel carrying the signal.

The Golden Code is a space-time code for Multiple-Input Multiple-Output (MIMO) systems. It accomplishes both the full rate and the full diversity. Its Bit Error Rate (BER) performance is the best compared to other 2×2 space-time codes till the date. Moreover, it has a non-vanishing coding gain thus it achieves the optimal diversity-multiplexing performance presented in. Because of those advantages, the Golden Code has been included into the 802.16e Wi-MAX standard. Hence, the decoding complexity is proportional to M\textsuperscript{4}, denoted by O(M\textsuperscript{4}). To reduce the decoding complexity of the Golden Code, several techniques have been proposed which gives better performance than V-Blast.

After the brief overview of MIMO and Golden code in section 2 and 3, section 4 depicts the proposed system. Section 5 illustrates Golden Coded Beamforming and its function in MIMO. Finally, Response of STBC system and conclusion are provided in section 6.

2 MIMO OVERVIEW

Multiple input multiple-output (MIMO) systems are a natural extension of developments in wireless communication. In MIMO networks, the system capacity is enhanced by scheduling multiple users to simultaneously share the spatial channel and hence the spatial degrees of freedom offered by multiple antennas has advantageously exploited. A MIMO technique has several key advantages [1] hence performance has begun to be intensely investigated.

- MIMO schemes allow for a direct gain in multiple access capacity proportional to the number of base station (BS) antennas.
MIMO appears more immune to most of propagation limitations plaguing single user MIMO communications such as channel rank loss or antenna correlation. Line of sight propagation, which causes severe degradation, is no longer a problem in multiuser setting. MIMO allows the spatial multiplexing gain at the base station to be obtained without the need for multiple antenna terminals, thereby allowing the development of small and cheap terminals.

The advantages of MIMO communication, which exploits the physical channel between many transmit and receive antennas, are receiving significant attention. While the channel can be so non-stationary that it cannot be estimated in any useful sense. The spatial diversity provided by multiple spatial paths reduces sensitivity to fading. Under certain environmental conditions, the power requirements associated with high spectral-efficiency communication can be significantly reduced by avoiding the compressive region of the information-theoretic capacity bound. At present, spectral efficiency is defined as the total number of information bits per second per Hertz transmitted from one array to the other array.

3 THE GOLDEN CODE

The Golden code was proposed for a 2 X 2 MIMO system as an optimum 2 X 2 linear dispersion space-time block codes. The code is constructed using cyclic division algebras which is a particular family of division algebras. It is built over a quadratic extension of the base field $Q(i)$, where $i^2 = -1$, thus use of arbitrary Q-QAM constellations is possible. Efficient constellation shaping is provided due to inherent integer lattice structure which is leading to the information lossless property. $Q(i)$ which is the base field provides the non-vanishing determinant (NVD) property for the Golden code, i.e., for any QAM size, the minimum determinant remains constant. It has been also proved that the NVD guarantees to achieve the fundamental performance limit of the multiple-input multiple-output (MIMO) systems, given by the diversity multiplexing tradeoff (DMT). The Golden code is optimal in several senses. It has:

- Full rate (2 symbols per channel use)
- Full diversity ($d = 4$)
- Non-vanishing determinant, it is clear that $\pm \min$ is independent of the constellation size.
- It preserved mutual information (unitarity of the vectorized codeword matrix).

3.1 Algebraic property

From [3] and [7], we review the algebraic property of the Golden code, which is developed using the cyclic algebra:

$$A = (L/K = Q(i, \sqrt{5})/Q(i), \sigma, \iota),$$

with $\sigma: \sqrt{5} \mapsto -\sqrt{5}$. The ring of integers of the field $L$ is given by,

$$\mathcal{O}_L = \{a + b\theta \mid a, b \in \mathbb{Z}[i]\}, \text{ where, } \theta = \frac{1+\sqrt{5}}{2}.$$  

Before shaping, a codeword from this algebra is given as

$$[a + b\theta \quad c + d\theta]$$

with four information symbols $a, b, c, d \in \mathbb{Z}[i]$. Since $i$ is not a norm of any element of $L$, $A$ is a cyclic division algebra [7]. By definition, the obtained codebook is linear, full rate since it contains four information symbols $a, b, c, d$ and fully diverse.

Following [6], we define a codeword $X$ belonging to the Golden code with the cubic shaping property,
\[
X = \frac{1}{\sqrt{5}} \begin{bmatrix}
\alpha (a + b\theta) & \alpha (c + d\theta) \\
i\sigma (a)(c + d\sigma(\theta)) & \sigma(a)(a + b\sigma(\theta))
\end{bmatrix}
\]

where \(a, b, c, d\) are QAM symbols.

When \(a, b, c, d\) can take any value in \(\mathbb{Z}[i]\), we have an infinite code \(C_\infty\). This terminology evokes the case where finite signal constellations are carved from the infinite lattices.

### 3.2 The Minimum Determinant

For the computation of the minimum determinant of the infinite code following steps are followed [6]. Since

\[
X = \frac{1}{\sqrt{5}} \begin{bmatrix}
\alpha & 0 \\
0 & \sigma(\alpha)
\end{bmatrix} \begin{bmatrix}
a + b\theta & c + d\theta \\
i(c + d\sigma(\theta)) & a + b\sigma(\theta)
\end{bmatrix}
\]

also \(\alpha \sigma(\alpha) = 2 + i\), hence,

\[
\det(X) = \frac{1}{2 - i} \left[ a^2 + ab - b^2 - i(c^2 cd - d^2) \right]
\]

By definition of \(a, b, c, d\), we have that the non trivial minimum of \(|\left[ a^2 + ab - b^2 - i(c^2 cd - d^2) \right]|\) is 1, thus

\[
\delta_{\text{min}}(C_\infty) = \min_{X \neq 0} |\det(X)| \geq \frac{1}{5}
\]

Thus the minimum determinant of the infinite code is bounded away from zero, as per the NVD property requirement. Finally, we note that in the second row of the codeword \(X\) the factor \(I\) guarantee a uniform average transmitted energy from both antennas in both channel uses, since \(|i|^2 = 1\).

### 4 METHODOLOGY

There are two space time codes: spatial division multiplexing (SDM) and the Golden code, for \(2 \times 2\) MIMO configuration.

![BICM STBC system](image)

**Figure 1. BICM STBC system [3]**

The block diagram of the proposed system is illustrated in Figure 1. The binary information elements \(b\) is the input to the transmitter. These elements are first encoded by a binary code of rate \(R_c\) e.g. a convolutional code. Output is then interleaved by a bit interleaver \(\pi\). The coded and interleaved sequence \(c\) is fed into the \(2^m\) QAM gray mapper and is mapped onto the signal sequence \(x \in X\). The resulting symbols are coded by an algebraic space time block code with spreading factor \(s\) and code generator matrix \(G\).

#### 4.1 Transmitter

Here we have used the vectorized notations for simplicity. For the \(2 \times 2\) configuration, we focus on two space time codes: spatial division multiplexing (SDM) and the Golden code, for \(2 \times 2\)
MIMO configuration. In this the SDM case corresponds to \( s = 1 \) and \( G = I_{n_t} \). In the Golden code case, \( s = 2 \) and the vectorized generator matrix is given by,

\[
G = \begin{pmatrix}
\alpha & \alpha \theta & 0 & 0 \\
0 & 0 & i \overline{\alpha} & i \alpha \theta \\
0 & 0 & \alpha & \alpha \theta \\
\bar{\alpha} & \alpha \theta & 0 & 0
\end{pmatrix}
\]

Where, \( \theta = \frac{1+\sqrt{5}}{2} \), \( \bar{\theta} = \frac{1-\sqrt{5}}{2} \), \( \alpha = 1 + i \theta \), \( \bar{\alpha} = 1 - i \bar{\theta} \) and \( \text{i} \) are QAM information symbols, \( i = 1 \cdots 4 \).

The coded codewords are finally transmitted on a multiple antenna channel \( H = [h_{i,j}] \) with \( n_t \) transmit antennas and \( n_r \) receive antennas; \( h_{i,j} \) denotes the Rayleigh fading coefficients between transmit antenna \( j \) and receive antenna \( i \) with \( h_{i,j} \sim \text{CN}(0, 1) \).

The bit interleaver can be modeled as \( \pi: k' \rightarrow (k, i) \), where \( k' \) denotes the original ordering of the coded bits \( C_{k'} \), \( k \) denotes the time ordering of the MIMO codewords \( X_k \) where \( X \in \mathcal{X}^{sn_t} \) and \( i \) indicates the position of the bits \( c_{k'} \) in the codeword.

### 4.2 Receiver

At the receiver, the vectorized received signal is given by,

\[
Y = H_e G X + Z
\]

where, \( Z \) is the complex Gaussian noise \( Z \sim \text{CN}(0, N_0 I_{n_t}) \) and \( H_e \) indicate the equivalent block diagonal channel. For the SDM case, \( H_e = H \), and for the Golden code

\[
H_e = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix}
\]

### 4.3 The ML soft decoder and Viterbi decoder

In multiple-input multiple-output (MIMO) fading channels maximum likelihood (ML) detection is advantageous to achieve high performance, but its complexity raise exponentially with the spectral efficiency.

The ML soft decoder generates for each coded bit \( C_{k,i} \) two metrics: \( \lambda_{ck=0}^i \) and \( \lambda_{ck=1}^i \) Theses metrics correspond to the log-MAP computed over one codeword, and are given by :

\[
\lambda_{ck}^i = \log \sum_{X \in \mathcal{X}_k} P(Y | H_e, X)
\]

\[
\approx \min_{X \in \mathcal{X}_k} \| Y - H_e G X \|^2
\]

where \( \| . \|^2 = ( . )^H ( . ) \) is the Euclidean distance and \( \mathcal{X}_k^i \) denotes the constellation subset.

\[
\mathcal{X}_b^i = \left\{ X \in \mathcal{X}^{sn_t} = X \times ... X: i^j (X) = b \right\}
\]

and \( i^j (X) \) is the \( j \)th bit of the codeword \( X \). For low complexity algorithms, these metrics can be work out using the list sphere decoder.

Then, the metrics associated to the interleaved bits are deinterleaved. Finally, the \( \lambda \) metrics are used by the Viterbi decoder to decode the information bits by verdicting the shortest path in the trellis according to,

\[
\hat{C} = \arg \min_{c \in C} \sum_{k'} \lambda (C_k^i)
\]


5 GOLDEN CODED MULTIPLE BEAMFORMING

The method of Golden Coded Multiple Beamforming (GCMB) coalesce the Golden Code with 2×2 multiple beamforming. GCMB achieves both the full rate and the full diversity similar to the general MIMO systems employing the Golden Code and 2 × 2 FPMB. All these methods have almost the same BER performance. However, the worst-case decoding complexity of GCMB is reduced to O(√𝑀) compared to general MIMO systems using the Golden Code. This complexity is lower than FPMB, whose worst-case complexity is O(M).

In the GCMB structure the information bit sequence is modulated by the M-ary square QAM. Consider four consecutive modulated complex-valued scalar symbols are given as $S_1, S_2, S_3$ and $S_4$.

The codewords $X$ of the Golden Code are $2 \times 2$ complex-valued matrices, given as

$$X = \frac{1}{\sqrt{5}} \begin{bmatrix} (1 + i\beta)S_1 + (\alpha - i)S_2 & (1 + i\beta)S_3 + (\alpha - i)S_4 \\ (i - \alpha)S_3 + (\alpha - i)S_4 & (1 + i\alpha)S_1 + (\beta - i)S_2 \end{bmatrix}$$

Where $\alpha = \frac{1 + \sqrt{5}}{2}, \beta = \frac{1 - \sqrt{5}}{2}$.

Assumed MIMO channel have two transmitting antennas and two receiving antennas. This MIMO channel is assumed to be quasi-static, Rayleigh, and flat fading. The $X$ for the mentioned configuration is set of complex numbers. The beamforming vectors are determined by the SVD of the MIMO channel,

$$i.e., H = U \Lambda V^H$$ (7)

where, $U$ and $V$: unitary matrices,

$\Lambda$: a diagonal matrix whose $s^{th}$ diagonal element, $\lambda_s \in \mathbb{R}^+$ is a singular value of $H$ in decreasing order,

$\mathbb{R}^+$: The set of positive real numbers

When S streams are transmitted at the same time, the first S vectors of $U$ and $V$ are chosen to be used as beamforming matrices at the receiver and the transmitter, respectively. In the case of GCMB, the number of streams $S = 2$. The received signal is

$$Y = \Lambda X + N, \quad (8)$$

where $Y$ is a $2 \times 2$ complex-valued matrix, and $N$ is the $2 \times 2$ complex-valued additive white Gaussian noise matrix whose elements have zero mean and variance $N_0 = S/SNR$. The channel matrix $H$ is complex Gaussian with zero mean and unit variance. The total transmitted power is scaled as $S$ in order to make the received Signal-to-Noise Ratio (SNR) SNR.

Let $\mathcal{X}$ denote the signal set of M-QAM. After the ML decoding we obtained,

$$\hat{x} = \min_{y \in \mathcal{X}} \| Y - \Lambda X \|^2$$ (9)

where $j \in \{1, \cdots, 4\}$. 
6 RESULT

![Response of V-BLAST system](image1)

Figure 2. Response of V-BLAST system [4]

![Response of STBC system](image2)

Figure 3. Response of STBC system

From figure 2 and 3, BER and SNR parameters of STBC system have better response. Hence in this paper, the STBC system is used. MIMO-OFDM with STBC is known as BICM (Bit Interleaved Coded Modulation). Here BICM with convolution code i.e. Golden code is used in MIMO system. Convolution code can be used for encoding of input binary stream followed by Square M-QAM modulator to reduce decoding complexity. Space-Time Block Coding (STBC) or/and Cyclic Delay Diversity (CDD) can be useful to map the spatial streams on different transmit chains. At the receiver ML soft decoder and error detection code i.e. viterbi code are used for detection. Since
each Golden Code codeword employs four information symbols from an M-QAM constellation, M^4 points are calculated by exhaustive search to achieve the Maximum Likelihood (ML) decoding. Complexity is reduced to 45% using golden code in MIMO system. The proposed system aims to achieve better performance than SDM (uncoded MIMO scheme) in a BICM-MIMO system.

REFERENCES